**Pretzels, Tiles, and Beyond!**

This lesson is intended for an Integrated Algebra I course learning about simplifying binomials and polynomial expressions. Students will work with algebra tiles and pretzels. Students will participate in group and class projects and are expected to contribute during the guided worksheet examples. This lesson is intended for an 80 minute class period.

**Standards:**

*NYS MST Standards/Performance Indicators:*

* A.PS.2 Recognize and use equivalent representations of a problem situation or a mathematical concept
* A.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving
* A.RP.8 Support an argument by using a systematic approach to test more than one case
* A.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work
* A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
* A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
* A.A.13 Add, subtract, and multiply monomials and polynomials

*2000 NCTM Principles and Standards in School Mathematics:*

* Algebra Strand: Draw reasonable conclusions about a situation being modeled.
* Communication Strand: Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
* Representation Strand: Use representations to model and interpret physical, social, and mathematical phenomena.

**Instructional Learning Outcomes:**

At the closure of the lesson, students should be able to:

* Simplify binomials and polynomials using hands-on models.
* Connect the hands on activities with previous properties.
* Express their ideas coherently in a group to complete various projects.

**Materials:**

1. Scratch off sheets (one for every three people) 4. Worksheets for bell ringers and guided notes
2. Algebra tiles (one set for groups of three) 5. Overhead tiles
3. Graph printed overheads 6. Pretzels

**Instructional Protocol:**

When the students walk into the classroom they will pick up a scratch off sheet, a worksheet for their bell ringer and a packet containing their guided notes. For this bell ringer, the students can use their calculators to work on the ten questions on their own for four minutes. Next, the students will be placed into random groups of three for five minutes to discuss and solve all ten questions. Students should hold on to all their bell ringer work as it is part of their ticket out the door. After the bell ringer, students should begin their guided notes. Students should be participating and following along during the guided notes. After the guided notes on how to use pretzel manipulatives, the students will be given pretzels. Groups of three should receive one set of pretzels. From here, the teacher should go through some examples with pretzels and have the students work in groups to solve some problems on their own. This should last approximately thirty minutes. Now students will turn in their pretzels and work with algebra tiles instead. Students will continue with guided notes while working with the algebra tiles. This should last approximately thirty minutes as well. Solutions to the guided practice can be found on the teachers copy in red. Then students will begin their IF-AT work in a group of three. This will serve as their ticket out the door. If it is not completed than, it is to be finished for homework.

**Bell Ringer**

Simplify the following expressions. Calculators are allowed.

**Bell Ringer Solutions**

6

-6

8

106

-24

31

44

17

47

157

**Math with Food!**

When simplifying polynomials involving addition and subtraction we can use items such as pretzels.

 The Square Pretzel will represent , because it’s sides are of length x.

 The Stick Pretzel will represent, because its side is length x and its width is length 1.

In this picture we have

The Twist Pretzel will represent our one unit(s).

If we’re using pretzels for our positives, how will we represent our negative values? How about chocolate covered pretzels?

 These will be our representation.

 These will be our representation.

 These will be our negative one unit(s).

Here is your task, use the pretzels to solve the following problems:

This multiplication expression cannot be done with pretzels because we cannot set pretzels up in an area format.

Perhaps there is something that we can use, (hands on) that we can set up in an area format. At this point students will have Algebra Tiles distributed to them. Each group of three will receive their own set of Algebra Tiles.

**Seeing is Believing**

Algebra Tiles were developed by Peter Rasmussun. With Algebra Tiles students are able to use a hands-on manipulative to simplify a polynomial expression. Here are a few things that you should know about Algebra Tiles before we begin.

 The large square represents , because the square’s sides are of length x.





The rectangle represents , because the length is x and its width is one unit.





 The small square represents one unit.



The large black square represents



 The black rectangle represents



The small black square represents units.

From our pretzel activity, we know that simplifying polynomial expressions with addition and subtraction is the same process for Algebra Tiles.

Can we multiply with Algebra Tiles? What if we set them up just like a multiplication table? Let’s try it!

Using Algebra Tiles, multiply:



\*







Using Algebra Tiles, we set one component of the expression vertically and the other component of the expression horizontally such that it forms a grid pattern. Now we can fill in the middle. Think about when you extend lines on drawings. Here if you extend lines from the components of our original expression you will obtain the simplification within the chart. After adding up all similar terms inside our chart we will see our value of is

Try this expression within your group. Make sure you notice that there are positive and negative numbers (Remember your properties of multiplying positive and negative numbers!)





\*







Through combining like terms and zero pairs, we have our final solution of:

What if you are given a product? Would you be able to determine the two expressions being multiplied together? How about if we start with the product and the black outlined portion below?

\*

If we begin with and the diagram in black we can place the Algebra Tiles into a square format and draw in our grid format. From our gird format, we can determine that we need to have to produce our answer. Think of this process as the reverse of our multiplication process we worked with earlier. (Here the blue represents the solution to the problem)

Now try this one on your own:

Solution: We can see that in order to obtain we would have .

**Homework**

Along with finishing your four pretzel questions from class, each student should look around their house (or on the internet) to determine what else they could use in place of Pretzel Manipulatives. You must find at least one example and explain why it would or would not work. You may not use pretzels as your example. The item could be food related or not, this is open to your interpretation.

Examples to expect on homework:

* Cookies
* Screws vs. Nails
* Balls – Basketball, softball, tennis ball, golf ball, …

With your new found hands-on manipulative complete the following exercises:

**Wrap Up/Conclusion**

Students complete the ticket out the door to reinforce the lesson.

**Ticket out the Door**

Simplify the following expressions.

**Ticket out the Door Solutions**

By grouping our like terms together we have:

Then combining the like terms we obtain:

Which we can see was choice (d)

Distribute the positive 2 through the parentheses:

Distribute the through the expression:

Which we can see is choice (b)



Following the same pattern as in question (1) we can determine that can be simplified to which is choice (a)



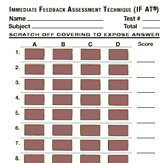
Reducing our fractions we can follow the same process as in questions (1) and (3) and obtain choice (c) which is

Remember from previous units that ! represents a factorial. This means that and . After determining the value of our factorials we can combine like terms to obtain which is choice (a)

For the remainder of the problems we can repeat the same process as above to obtain the following answers. (Don’t forget that if we are subtracting a large quantity then we need to distribute the negative throughout the quantity).

1. (d)
2. (b)
3. (a)
4. (c)
5. (b)

During the bell ringer, the instructor should be monitoring the students to determine that they are doing the IF-AT. The following should be seen on the scratch off cards during the IF-AT process.



A plain scratch off card before the students have checked their answers.

The point system for the IF-AT scratch off cards is as follows:

If students find the correct solution the first time, they receive four points.

If the students find the correct solution on the second attempt, they receive two points.

If the students find the correct solution on their third try, they will receive one point.

If the students do not find the correct solution after three attempts, they will receive no points for that question.



A completed scratch off sheet. The instructor should go over any questions that the students had difficulty finding the answer.

Teachers who are interested in the IF-AT or would like to purchase IF-AT scratch off sheets should visit

<http://www.epsteineducation.com/home/about/default.aspx>. Teachers should be aware though; for 10 questions with four different choices the IF-AT scratch off sheets cost $80 for their smallest available package of 500 cards.

Sometimes it is easier to simplify polynomials using a hands on method. When using Algebra Tiles keep in mind that you need to remember your basics like a negative multiplied by a positive is a negative.

For more information on Algebra Tiles and how to incorporate fractions with Algebra Tiles, visit the National Library of Virtual Manipulatives online at <http://nlvm.usu.edu/en/nav/vLibrary.html>.

For a refresher on pretzel manipulatives, go to <http://www.youtube.com/watch?v=P51MfTL1KTs> .

Remember, algebra is difficult for many students to understand. With the assistance of a hands- on manipulative there is a larger chance that students will be able to understand algebra.

Homework will be collected at the beginning of next class while students are working on their bell ringer.

When the bell rings for students to leave class, they will need to turn in their IF-AT scratch off card and the work accompanying it. This is how each student will receive their participation grade.